Semantic Decomposition for Solving Distance Constraints

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Numerical constraint problems occurs in numerous applications. Consistency techniques on finite domains have been adapted to handle continuous CSP(e.g. 2B-consistency, kB-consistency[1], Box-consistency[2], ...). Roughly speaking, these local filtering methods compute an external approximation of the solution space. That is to say, intervals the bounds of which are local consistents. Thus, this external approximation still contains a huge number of local inconsistent values. Splitting techniques are often used to isolate individual solutions. However, these techniques are ineffective when the domain contains continuous subspaces of solutions. Moreover, they do not take advantage of the semantic of the constraints for splitting the intervals.

We introduce here a pruning technique, called LDF(Local Decomposition Filtering)[3], for solving systems of distance equations. This method is based on a local decomposition of the domains which is guided by the properties of the distance constraints(convexity and monotonicity). More precisely, the domains of the coordinates of two points involved in a constraint c is decomposed using the properties of c(see figure 1). The canonical form of the distance equations($X^2 + Y^2 - D^2 = 0$) identifies the monotonous and convex parts on $\mathbb{R}^+ \times \mathbb{R}^+$, $\mathbb{R}^+ \times \mathbb{R}^-$, $\mathbb{R}^- \times \mathbb{R}^+$ and $\mathbb{R}^- \times \mathbb{R}^-$.

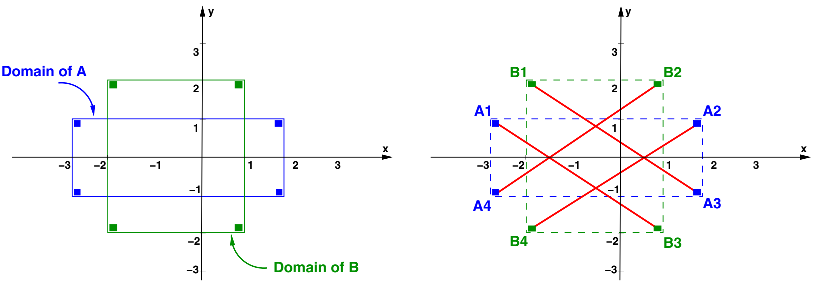

Fig. 1. Consider constraint $(x_A - x_B)^2 + (y_A - y_B)^2 = D^2$ with $D \in [4.95, 5]$. Left picture shows the domains of the points A and B after 2B-consistency filtering. Right picture shows the four subdomains of A and B and the micro-structure computed by LDF.

[1] O. Lhomme. *Consistency techniques for numerical CSPs*. Proceedings of the 13th IJCAI. Chambéry, France, August 28 -September 3, 1993. Morgan Kaufmann, p232-238.

[2] P. Van Hentenryck, D. McAllester, and D. Kapur. *Solving Polynomial Systems Using a Branch and Prune Approach*. SIAM Journal on Numerical Analysis , 34(2), p797-827, April 1997.

[3] H. Batnini and M.Rueher. *Filtrage local par décomposition de CSP continus*. JNPC'03. 9eme Journées Nationales pour la résolution de Problemes NP-Complets. p39-51. Juin 2003. Amiens. France.

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This decomposition is propagated on the other variables by the way of a local consistency algorithm. A graph of intervals is updated by a specific projection procedure which connects the consistent pairs of subdomains. We prove that the global reduction of the domains is at least equivalent with that carried out by *2B-consistency*. The structure of the resulting graph can be compared to an arc-consistant finite CSP. Thus, classical searching algorithms can be used to identify potential subspaces of solutions.

The first results on academic examples are promising. Further work concerns experimentations on real problems (robotics and theory of the mechanisms) as well as the extension of this approach to other systems of non-linear constraints.