Segmentation

University of the Philippines

August 2006

Diane Lingrand

University of Nice – Sophia Antipolis, France

lingrand@polytech.unice.fr

http://www.polytech.unice.fr/~lingrand

Why edge detection ?

- in order to detect an object in a scene
  - Example: a robot that wants to play soccer
- in order to do measures
  - objects verification in a factory
- in order to extract informations
- in order to compress image data

Which primitives ?

- Edges
- Regions
- Point of interest or corners
- Patterns

Course's overview

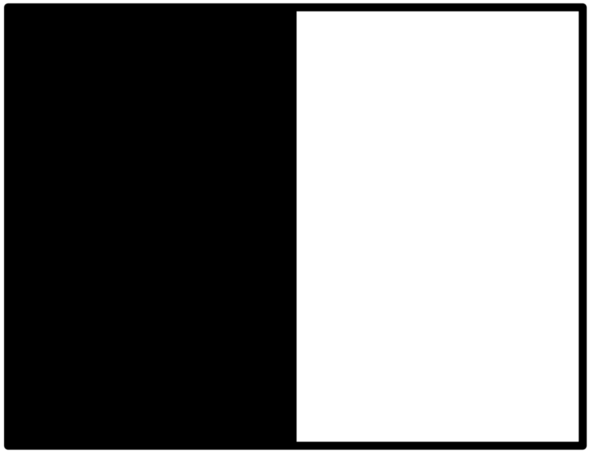
- Edge detection
  - A simple example
  - Convolution approach
    - 1^{st} derivative (Sobel, Kirsch, Prewitt, …)
    - 2^{nd} derivative (Laplacien, …)
  - Optimal filtering
    - Canny, Deriche
- Regions detection
  - similarity
  - Hough transform
  - contour deformation

Where are the edges ?


PIÉTONS
ATTENTION
TRAVERSEZ
EN 2 TEMPS

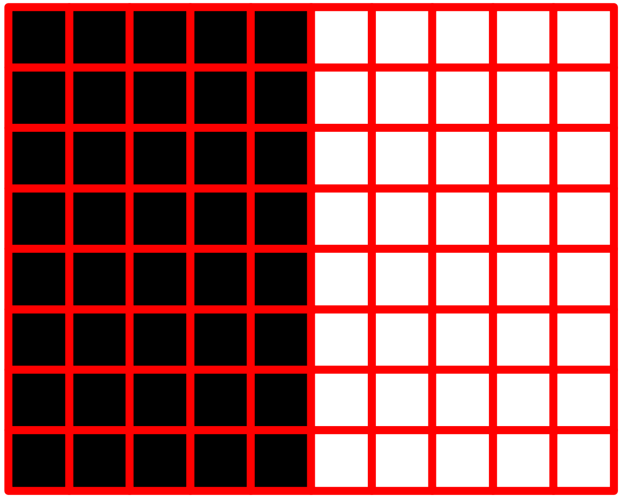
Where are the edges ? (2)

- Let us begin with a very simple image :



Where are the edges ? (3)

- Let us look closer to the pixels :

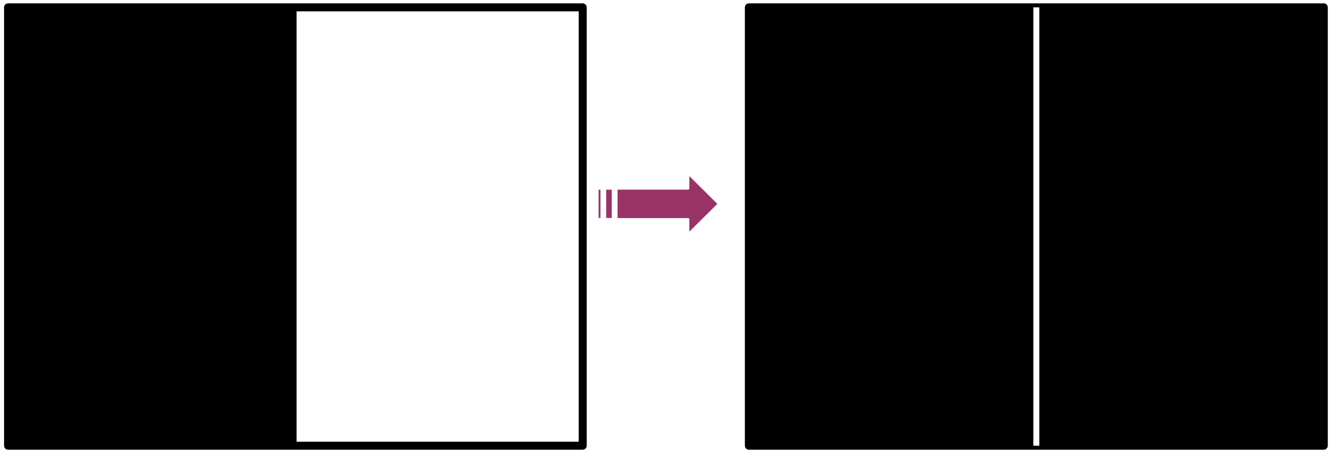


A first filter (1)

- Let us build an new image where each pixel (i,j) is obtained from the difference between pixel at (i,j) and pixel at (i-1,j)
  - pixels are still between 0 and 255: we need to transform the results
  - positive and negative differences: we only need the absolute value
  - be careful of image borders !

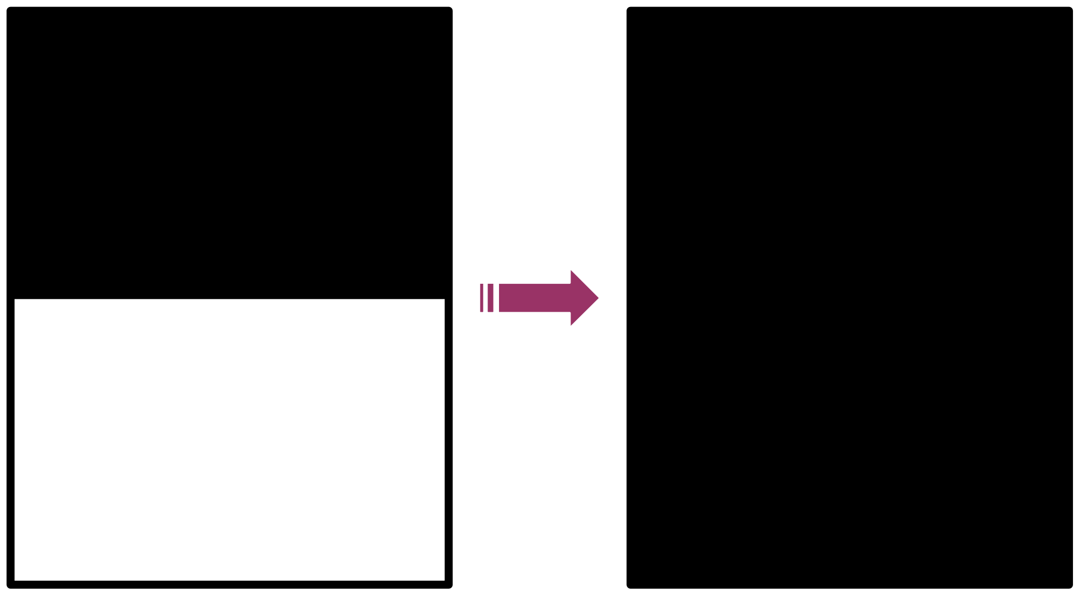
A first filter (2)

- Result on our simple image



A first filter (3)

- Result on another image



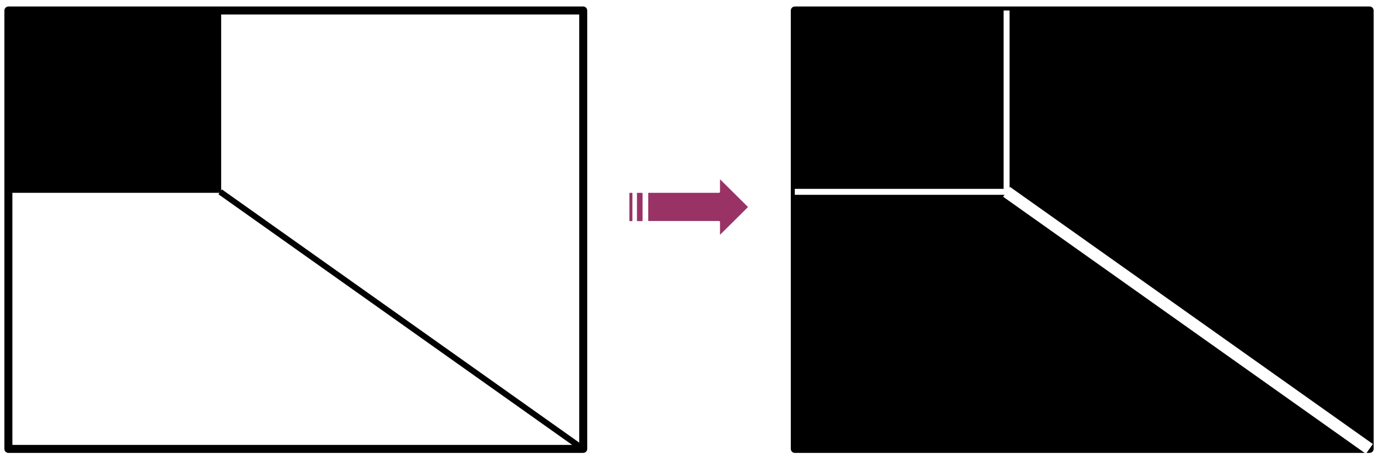
Wrong !

Enhancing our first filter

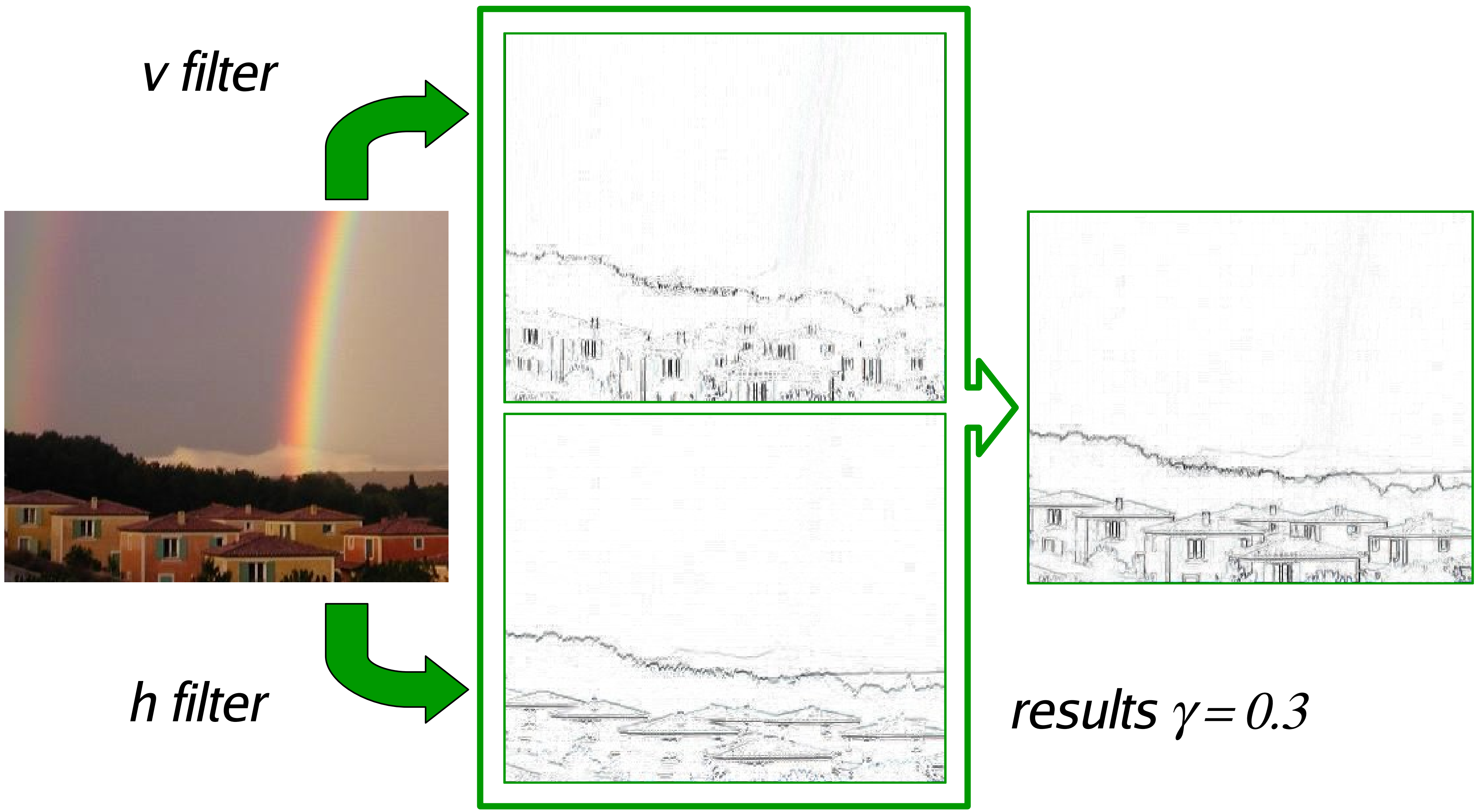
- We will compose :
  - a vertical edge detector
    - difference between (i,j) and (i-1,j) :
      - $I_{cv}(i,j) = I(i,j) - I(i-1,j)$
  - and an horizontal edge detector
    - difference between (i,j) and (i,j-1)
      - $I_{ch}(i,j) = I(i,j) - I(i,j-1)$
  - magnitude : $I_c = \mathrm{sqrt}(I_{cv}(i,j)^2 + I_{ch}(i,j)^2)$

Let us try on a single image

- Image without noise :



Let us try on a real image



More in details

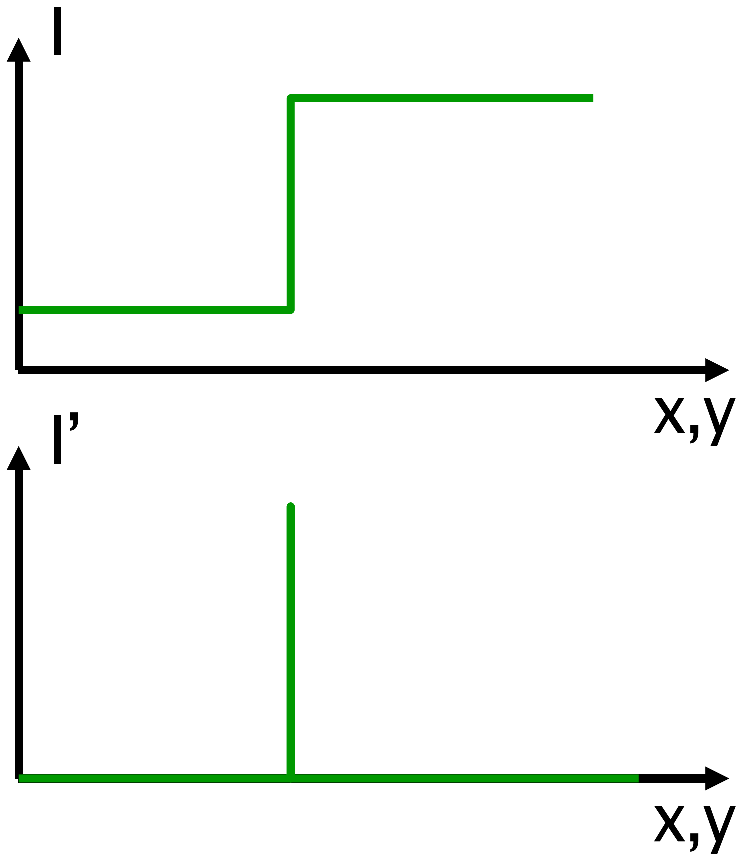
Other image :

Conclusion on our first filter

- Drawbacks:
  - sensitive to noise
  - too many edges detected
  - more sensitive to horizontal and vertical edges than the others
- Advantages:
  - fast
  - not difficult to implement

The ideal edge

- edge = where the intensity is not continuous
- maxima of the derivative



Intensity derivative

- Definition: $$\frac{df}{dx}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

with h at a maximum of 1

- Approximation :

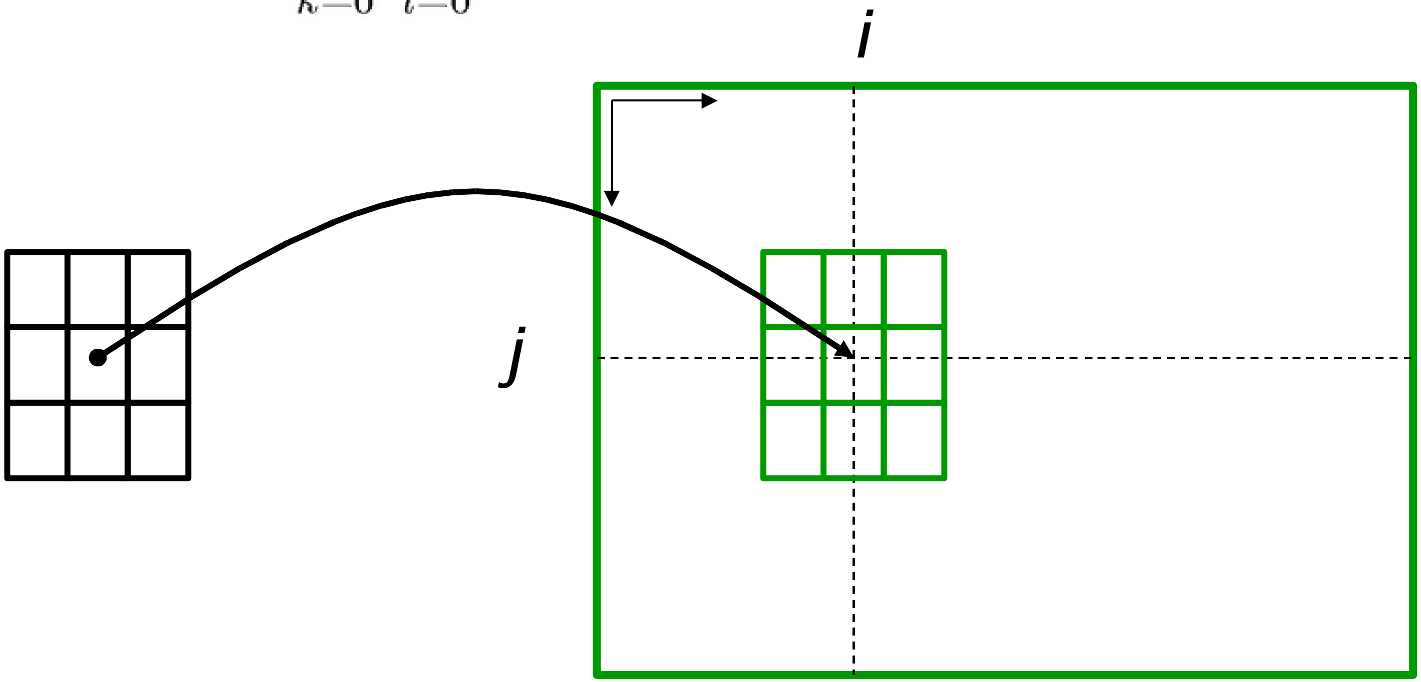
$$\frac{\partial f}{\partial x} = I[x+1] - I[x]$$

! *that is not really correct*

Convolution

- Convolution operator with a kernel K

$$I_2(i,j) = \sum_{k=0}^{2} \sum_{l=0}^{2} I_1(i+k-1, j+l-1) K(k,l)$$



Convolution in Java

```
float [] filtre = {-1.0f, 0.0f, 1.0f,
                   -2.0f, 0.0f, 2.0f,
                   -1.0f, 0.0f, 1.0f };
Kernel kernel = new Kernel(3, 3, filtre);
ConvolveOp cop = new ConvolveOp( kernel,
  ConvolveOp.EDGE_NO_OP, null);
cop.filter(biInput,biOutput);
```

sign

or:

ConvolveOp.EDGE_ZERO_FILL

Roberts filters (1965)

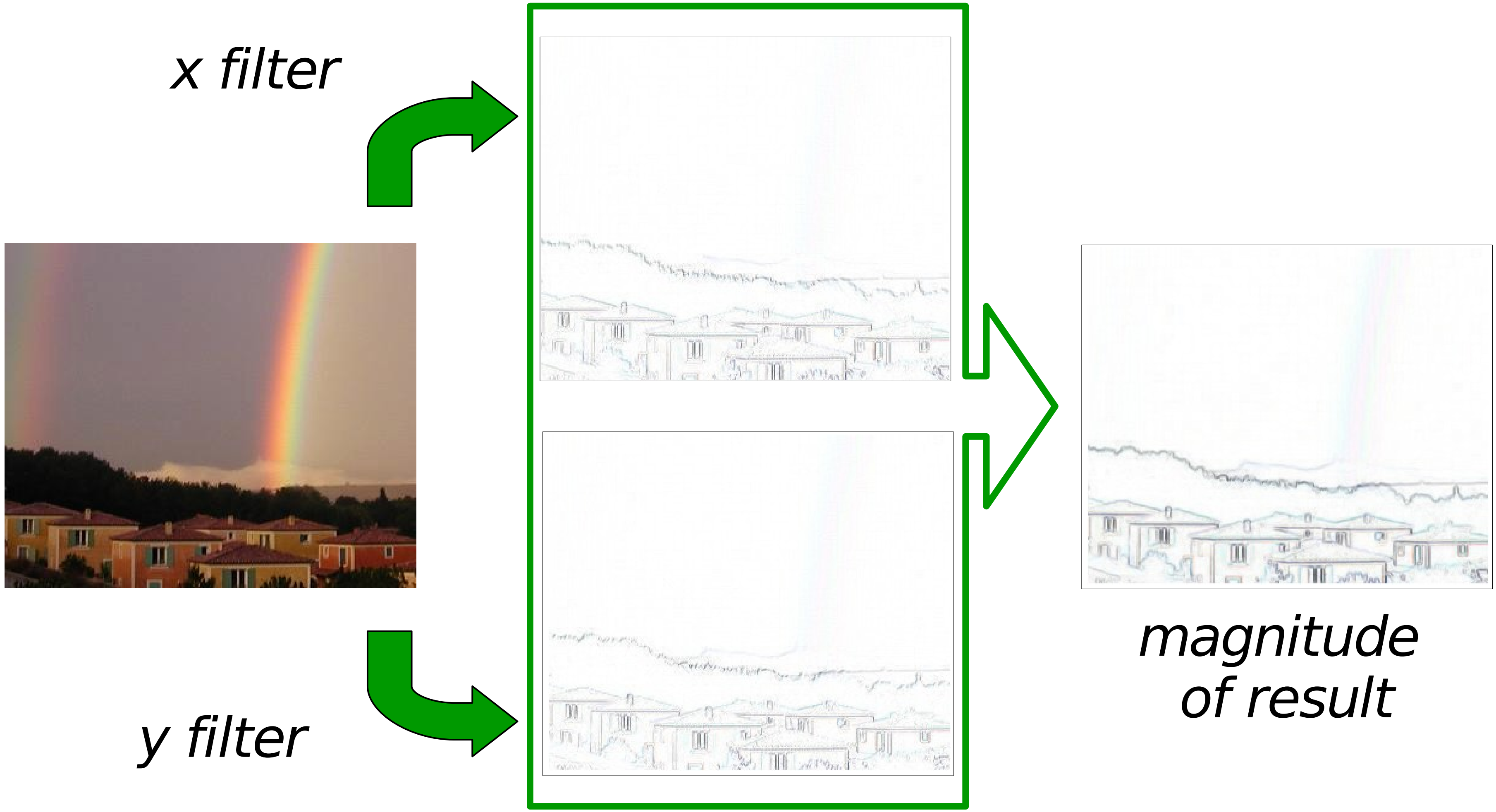
$$\frac{dI}{dx} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\frac{dI}{dy} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

- Magnitude = edge strength $\sqrt{(\frac{dI}{dx})^2 + (\frac{dI}{dy})^2}$ *ou* $\max(\left|\frac{dI}{dx}\right|, \left|\frac{dI}{dy}\right|)$

- Direction of edge normal $\arctan((\frac{dI}{dy})/(\frac{dI}{dx}))$

Example



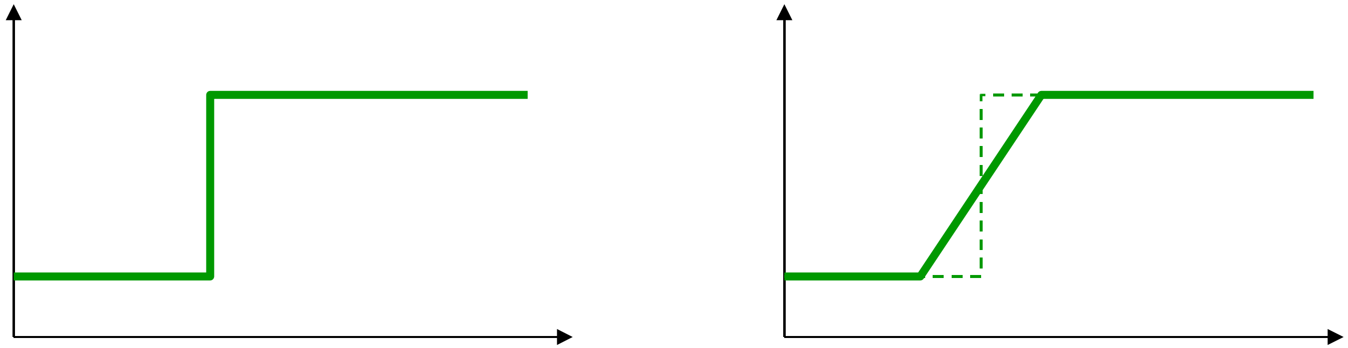
Sensibility to noise

- Let us assume : $I_{obs} = I_{signal} + \varepsilon \sin(\omega x)$
- Derivative : $I'_{obs} = I'_{signal} + \varepsilon\, \omega \cos(\omega x)$
- $\omega = 2\pi f$: high frequencies will disturb the signal derivative
- we need to eliminate these high frequencies : smooting
  - mean smoothing
  - gaussian smoothing
  - exponential filtering, ….

Mean smooting

- ex: filter 3x3

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} / 9$$



Gaussian smoothing (Marr, 1980)

- $h(x,y) = 1/(2\pi\sigma^2) \exp(-(x^2+y^2)/(2\sigma^2))$
- troncated and discretised gaussian :

0	1	0
1	4	1
0	1	0

/ 8

1	1	1
1	8	1
1	1	1

/ 16

Sobel filter (1970)

- Convolution :
  - smoothing [1 2 1]
  - derivative [1 0 -1]
- Mean of « derivatives » at x and x-1

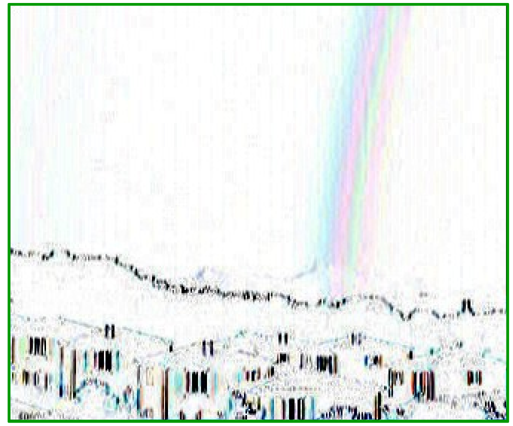
$$\frac{I[x+1] - I[x-1]}{2}$$

$$\frac{dI}{dx} = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{pmatrix} / 4$$

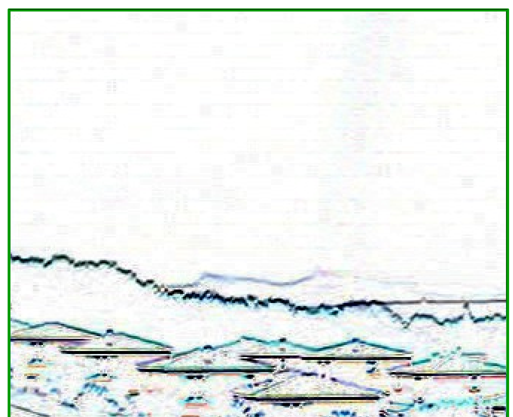
$$\frac{dI}{dy} = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix} / 4$$

Sobel : results (1)

v filter



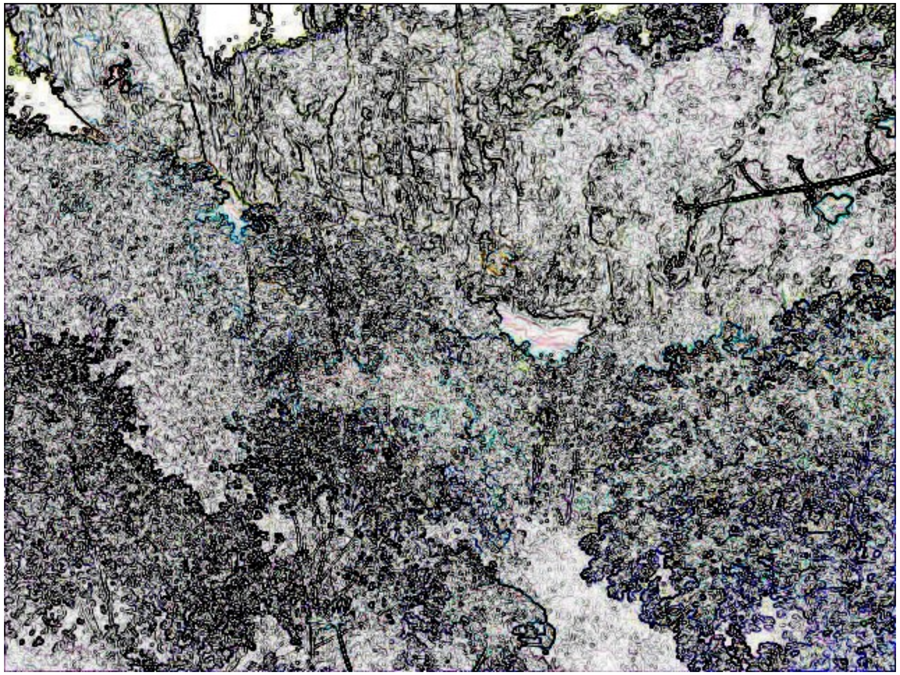
h filter



magnitude of result

Closer

Sobel : results (2)



Sobel : results (3)

Other filters : using more directions

- Prewitt (1970)

$$\begin{pmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{pmatrix} \ldots \; / \, 3$$

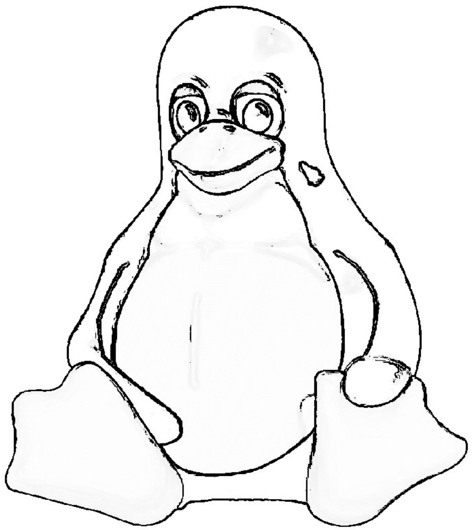
- Kirsch (1971)

$$\begin{pmatrix} 5 & -3 & -3 \\ 5 & 0 & -3 \\ 5 & -3 & -3 \end{pmatrix}, \begin{pmatrix} 5 & 5 & -3 \\ 5 & 0 & -3 \\ -3 & -3 & -3 \end{pmatrix}, \begin{pmatrix} 5 & 5 & 5 \\ -3 & 0 & -3 \\ -3 & -3 & -3 \end{pmatrix} \ldots \; / \, 15$$

Enhancement : thresholding

- we eliminate all the pixels that have a value below a minimum threshold

How to choose the threshold ?

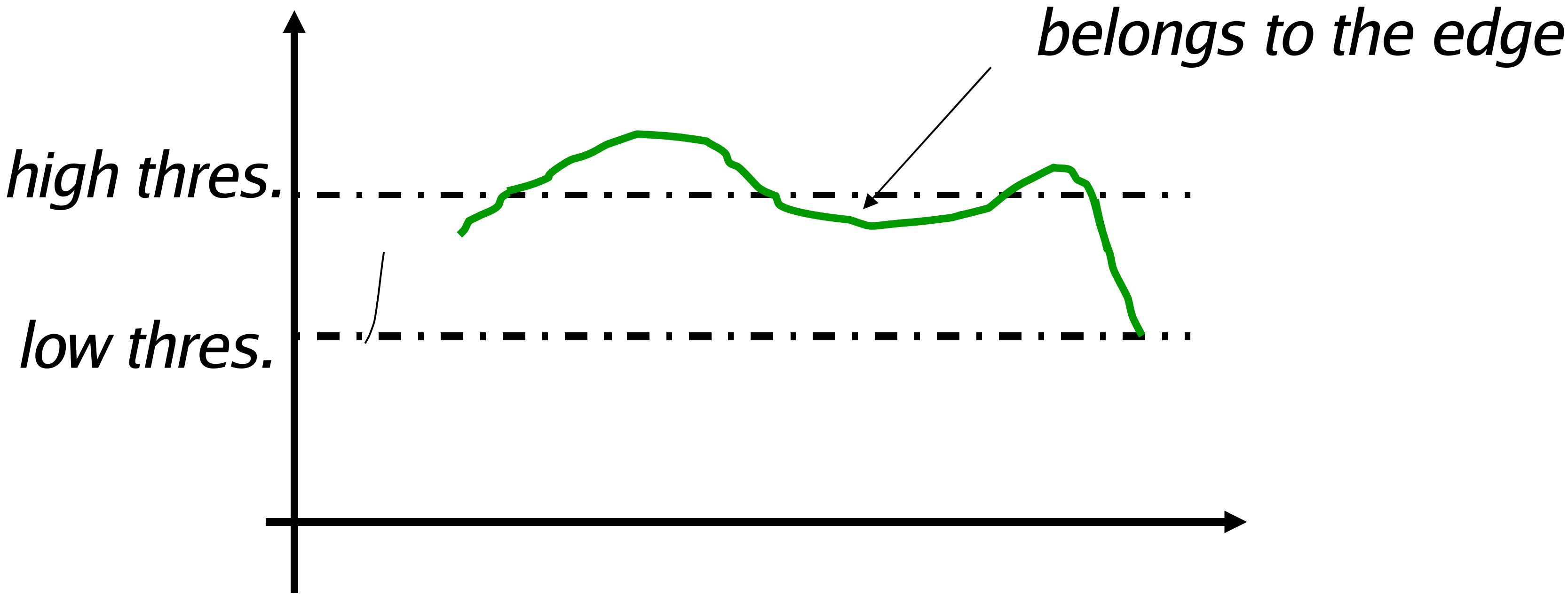


- Low threshold: all edges are detected but we have false positives

- High threshold: all the pixels detected are edge pixels but we are missing some of them (false negatives)

Enhancement (2)

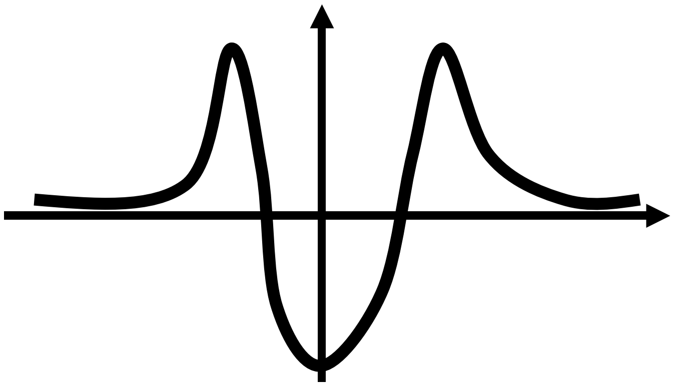
- Hysteresis thresholding



Laplacian

- Laplacian $\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$

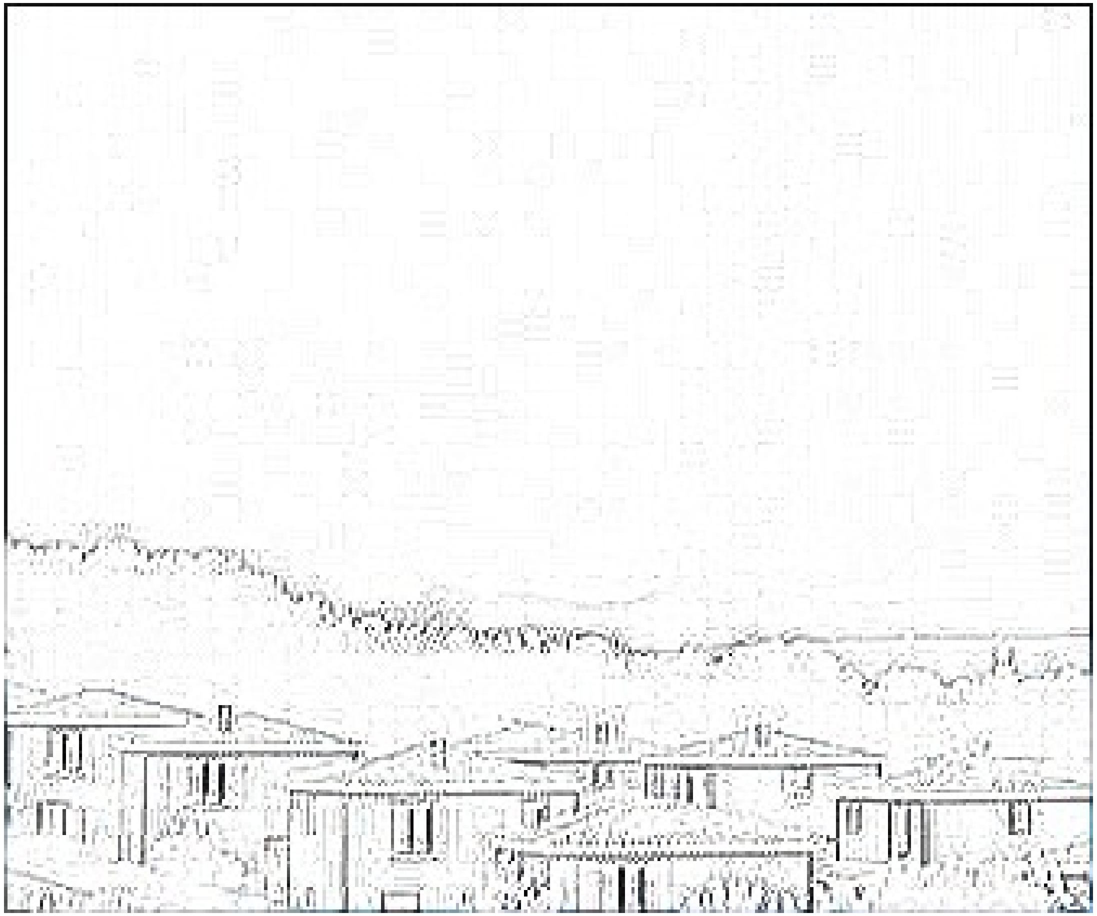
- Smoothing and Laplacian : Laplacian of a gaussian



$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{pmatrix} / 4$$

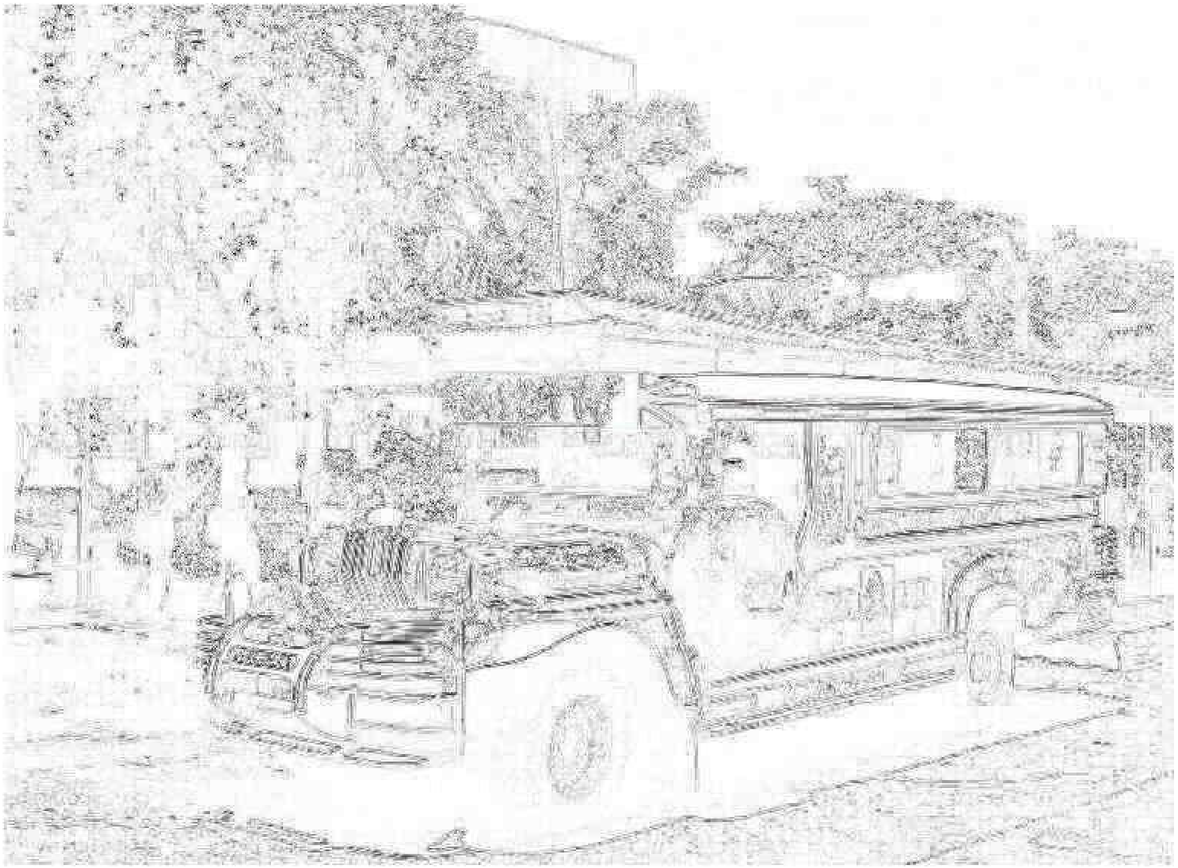
$$\begin{pmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{pmatrix} / 8$$

Laplace : results (1)



$\gamma = 0.3$

Laplace : results (2)

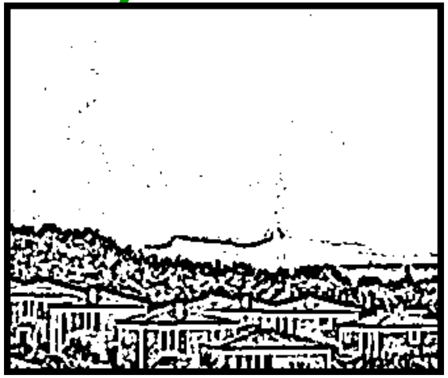


$\gamma = 0.3$

DOG Laplacian (80's) Marr and Hildreth

- Laplacian seen as the difference between two gaussian smoothing filters with different kernels
- Edge image : difference between a weakly smoothed image and a strongly smoothed image
  - try it on computers

Huertas-Médioni (1986)



- Laplacian of a gaussian :

$$\Delta g(x,y) = \frac{1}{G_0}\left(2 - \frac{x^2+y^2}{2\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- separation : $g_1(x).g_2(y) + g_1(y).g_2(x)$ with:

$$g_1(x) = \frac{1}{\sqrt{2G_0}}\left(1 - \frac{x^2}{2\sigma^2}\right)e^{-\frac{x^2}{2\sigma^2}}$$

$$g_2(x) = \frac{1}{\sqrt{2G_0}}e^{-\frac{x^2}{2\sigma^2}}$$

Example: 1/G0 = 4232 et $\sigma^2 = 2$:

g1 = [-1 -6 -17 -17 18 46 18 -17 -17 -6 -1]

g2 = [0 1 5 17 36 46 36 17 5 1 0]

What about color ?

- Detection on each color component (R, G, B) separately and recomposition of the color after filtering ?
- Max of the results on each component ?
- Mean ?
- Luminance (Y) ?

Exercise

- In file ConvolutionFilter.java
  - implement the convolution function

    public BufferedImage filter(BufferedImage bin)
- Try the following filters :
  - Sobel, Prewitt, Kirsch, Laplace
- Try different gaussian filters
  - using different kernels
- Combine filters, smootings et thresholding
  - keep the best results

```
public ConvolutionFilter(int type) {
        super();
        name = NAMES[type] + " filter";
        filters = new ArrayList();
        double [][] filter;
        switch(type) {
        case NAIVE_X:
          filter = new double [3][3];
          filter[0][0] = 0; filter[0][1] = 0; filter[0][2] = 0;
          filter[1][0] = -1; filter[1][1] = 1; filter[1][2] = 0;
          filter[2][0] = 0; filter[2][1] = 0; filter[2][2] = 0;
          filters.add(filter);
          break;
        case NAIVE_Y:
          filter = new double [3][3];
          filter[0][0] = 0; filter[0][1] = -1; filter[0][2] = 0;
          filter[1][0] = 0; filter[1][1] = 1; filter[1][2] = 0;
          filter[2][0] = 0; filter[2][1] = 0; filter[2][2] = 0;
          filters.add(filter);
          break;
        case NAIVE:
          filter = new double [3][3];
          filter[0][0] = 0; filter[0][1] = 0; filter[0][2] = 0;
          filter[1][0] = -1; filter[1][1] = 1; filter[1][2] = 0;
          filter[2][0] = 0; filter[2][1] = 0; filter[2][2] = 0;
          filters.add(filter);
          filter = new double [3][3];
          filter[0][0] = 0; filter[0][1] = -1; filter[0][2] = 0;
          filter[1][0] = 0; filter[1][1] = 1; filter[1][2] = 0;
          filter[2][0] = 0; filter[2][1] = 0; filter[2][2] = 0;
          filters.add(filter);

                    . . .
```

```
        case GAUSSIAN3:
          filter = GAUSSIAN3_FILTER;
          filters.add(filter);
          break;
        case GAUSSIAN5:
          filter = GAUSSIAN5_FILTER;
          filters.add(filter);
          break;
              . . .
        default:
        }
}
```

```
public  BufferedImage filter(BufferedImage bin) {
        int w = bin.getWidth();
        int h = bin.getHeight();
        BufferedImage bout = new BufferedImage(w,h,
                        BufferedImage.TYPE_INT_RGB);
        for(int i = 0 ; i < w; i++ )
          for(int j = 0 ; j < h; j++ )
                        bout.setRGB(i,j,0);

        int nbFilters = filters.size();

        switch(nbFilters) {
        case 0:
          return bout;
          break;
        case 1:
          .....
```

Optimal filtering

- Canny (1986)
- impulsional response h(x) such as:
  - good detection
  - good localisation
  - weak multiplicity of maxima due to noise
- we search for an edge
  - of magnitude A
  - with an additive gaussian noise of null mean and variance n_0^2

Quality 1: good detection

- The more smoothing, the better the quality of the detection
  - we want to maximize the ratio signal by noise :

$$\Sigma = \frac{A \int_{-\infty}^{0} f(x)dx}{n_0 \int_{-\infty}^{\infty} f^2(x)dx}$$

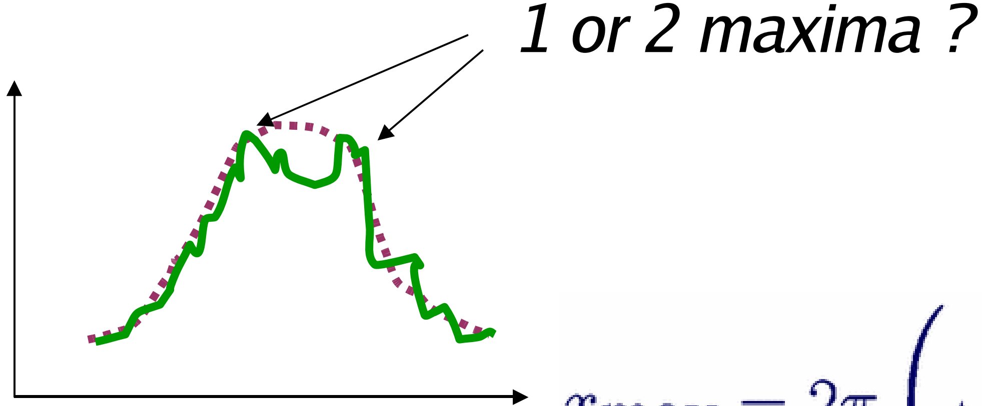
Quality 2: Localisation

- The less smoothing, the better the localisation.
  - we want to minimize the variance of the positions of roots. It is equivalent to maximize:

$$\Lambda = \frac{A|f'(0)|}{n_0 \int_{-\infty}^{\infty} f'^2(x)dx}$$

Quality 3: unique response

- We want an unique response for one edge : we limit the distance between 2 maxima



$$x_{\max} = 2\pi \left(\sqrt{\frac{\int_{-\infty}^{\infty} f'^2(x)dx}{\int_{-\infty}^{\infty} f''^2(x)dx}} \right)$$

General optimal filter

- Trade-off between Σ and Λ
- Find f maximising $\Sigma\Lambda$ under the constraint : $x_{max} = k$

which consist to solve the differential equation:

$$2f(x) - 2\lambda_1 f''(x) + 2\lambda_2 f''''(x) + \lambda_3 = 0$$

of general solution :

$$f(x) = a_1 e^{\alpha x} \sin(\omega x) + a_2 e^{\alpha x} \cos(\omega x) + a_3 e^{-\alpha x} \sin(\omega x) + a_4 e^{-\alpha x} \cos(\omega x)$$

Optimal filtering : Canny

- finite impulse response filter [-M;+M]
- constraints :

  $x_{max} = kM$

  $f(0) = 0$; $f(M) = 0$;

  $f'(0) = S$; $f'(M) = 0$;

  $x<0 : f(x) = -f(-x)$
- numerical optimisation : $\Sigma\Lambda = 1.12$
- approximation : derivative of a gaussian ($\Sigma\Lambda = 0.92$, $k = 0.51$, 20% lower performance)

Optimal filtering : Deriche (1987)

- infinite impulse response filtering
- constraints :

  f(0) = 0 ; f(+∞) = 0 ; f'(0) = S ; f'(+∞) = 0;

- where :

$$\boxed{f(x) = \frac{S}{\omega} e^{-\alpha|x|} \sin(\omega x)}$$

$$\Lambda = \sqrt{2\alpha} \qquad \Sigma = \sqrt{\frac{2\alpha}{\alpha^2+\omega^2}}$$

$$\Sigma\Lambda = \frac{2\alpha}{\alpha^2+\omega^2} \quad k = \sqrt{\frac{\alpha^2+\omega^2}{5\alpha^2+\omega^2}}$$

Optimal filtering: Deriche (2)

- Let us set $\alpha = m\omega$:

case 1	$m >> 1$	$\Lambda = \sqrt{2\alpha}$	$\Sigma = \sqrt{\frac{2}{\alpha}}$	$\Sigma\Lambda = 2$	$k = 0.44$
case 2	$m = 1$	$\Lambda = \sqrt{2\alpha}$	$\Sigma = \sqrt{\frac{1}{\alpha}}$	$\Sigma\Lambda = \sqrt{2}$	$k = 0.58$
case 3	$m = \sqrt{3}$	$\Lambda = \sqrt{2\alpha}$	$\Sigma = \sqrt{\frac{3}{2\alpha}}$	$\Sigma\Lambda = \sqrt{3}$	$k = 0.5$

<u>case 1</u> : the best, corresponds to $Sxe^{-\alpha|x|}$

<u>case 2</u> : for same value of k, performance index of Deriche increase of 25 %

<u>case 3</u> : for same value of k, performance index of Deriche increase of 90 %

Optimal filtering : Deriche (3)

- optimal integrator: $S(\alpha|x|+1)e^{-\alpha|x|}$
- bidimensionnal filters:

  $f_x(x,y) = k_1 m\, e^{-\alpha|m|}\, k_2(\alpha\ |n|+1)\, e^{-\alpha|n|}$

  $f_y(x,y) = k_1(\alpha\ |m|+1)\, e^{-\alpha|m|}\, k_2 n\, e^{-\alpha|n|}$

- recursive implementation

Recursive implementation

- Phase 1

for m from 0 to (w-1) :

for n from 0 to (h-1) :

$$y_1(m,n) = a_1 I_1(m,n) + a_2 I_1(m,n-1) + b_1 y_1(m,n-1) + b_2 y_1(m,n-2)$$

for n from (h-1) to 0 :

$$y_2(m,n) = a_3 I_1(m,n+1) + a_4 I_1(m,n+2) + b_1 y_2(m,n+1) + b_2 y_2(m,n+2)$$

for n from 0 to (h-1) :

$$r(m,n) = c_1(y_1(m,n) + y_2(m,n))$$

- Phase 2

for n from 0 to (h-1) :

for m from 0 to (w-1) :

$$y_1(m,n) = a_5 r(m,n) + a_6 r(m-1,n) + b_1 y_1(m-1,n) + b_2 y_1(m-2,n)$$

for m from (w-1) to 0 :

$$y_2(m,n) = a_7 r(m+1,n) + a_8 r(m+2,n) + b_1 y_2(m+1,n) + b_2 y_2(m+2,n)$$

for m from 0 to (w-1) :

$$I_2(m,n) = c_2(y_1(m,n) + y_2(m,n))$$

Coefficients for Deriche filters

	Smoothing	x-derivative	y-derivative	Laplace 1	Laplace 2
k	$\frac{(1-e^{-\alpha})^2}{1+2\alpha e^{-\alpha}-e^{-2\alpha}}$	$\frac{(1-e^{-\alpha})^2}{1+2\alpha e^{-\alpha}-e^{-2\alpha}}$	$\frac{(1-e^{-\alpha})^2}{1+2\alpha e^{-\alpha}-e^{-2\alpha}}$	$\frac{1-e^{-2\alpha}}{2\alpha e^{-\alpha}}$	$\frac{1-e^{-2\alpha}}{2\alpha e^{-\alpha}}$
a_1	k	0	k	1	0
a_2	$ke^{-\alpha}(\alpha-1)$	1	$ke^{-\alpha}(\alpha-1)$	0	1
a_3	$ke^{-\alpha}(\alpha+1)$	-1	$ke^{-\alpha}(\alpha+1)$	$e^{-\alpha}$	1
a_4	$-ke^{-2\alpha}$	0	$-ke^{-2\alpha}$	0	0
a_5	k	k	0	1	0
a_6	$ke^{-\alpha}(\alpha-1)$	$ke^{-\alpha}(\alpha-1)$	1	0	1
a_7	$ke^{-\alpha}(\alpha+1)$	$ke^{-\alpha}(\alpha+1)$	-1	$e^{-\alpha}$	1
a_8	$-ke^{-2\alpha}$	$-ke^{-2\alpha}$	0	0	0
b_1	$2e^{-\alpha}$	$2e^{-\alpha}$	$2e^{-\alpha}$	$e^{-\alpha}$	$2e^{-\alpha}$
b_2	$-e^{-2\alpha}$	$-e^{-2\alpha}$	$-e^{-2\alpha}$	0	$-e^{-2\alpha}$
c_1	1	$-(1-e^{-\alpha})^2$	1	1	$\frac{1-e^{-2\alpha}}{2}$
c_2	1	1	$-(1-e^{-\alpha})^2$	1	$\frac{1-e^{-2\alpha}}{2}$

Deriche smoothing

0.25

0.5

0.75

1

2

3

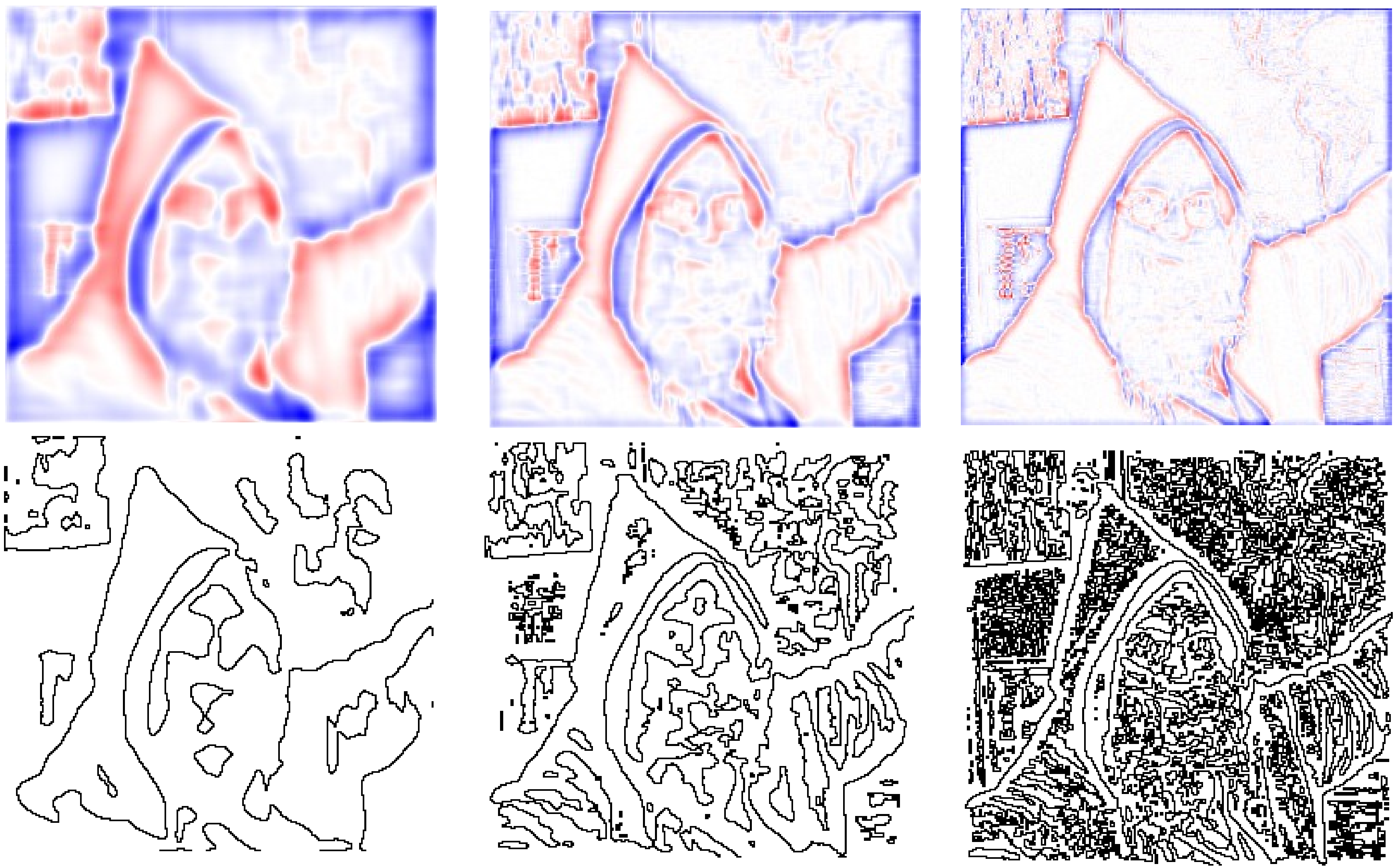
Edges with first derivative by Deriche

0.5

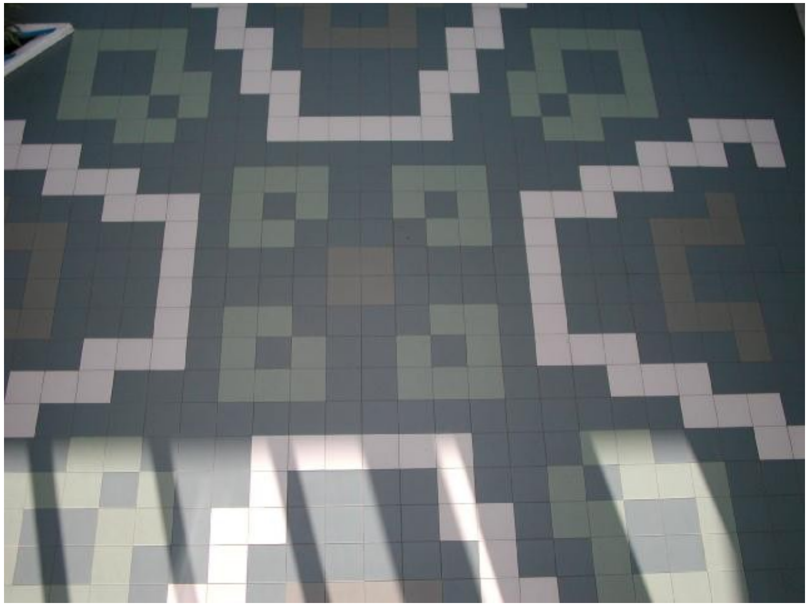
1

3

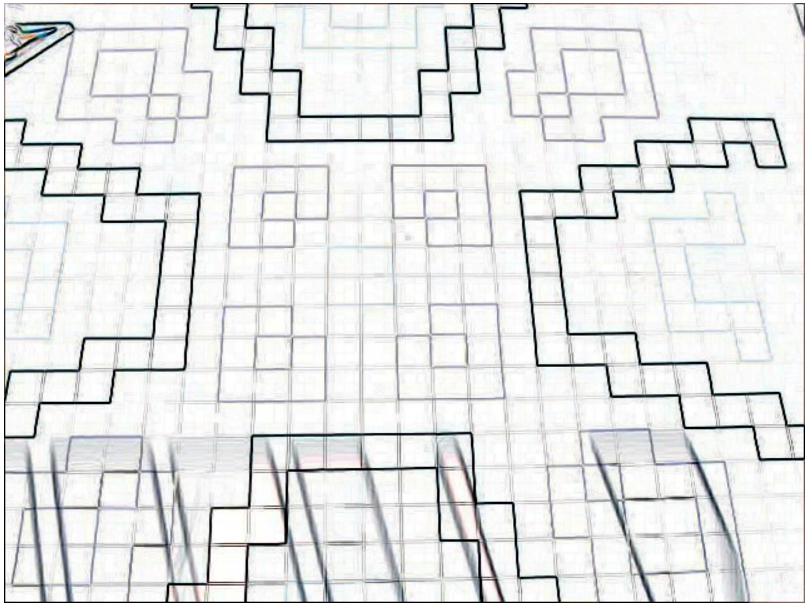
Laplace by Deriche : sign and zeros



An easy image



solEssi3212.jpg



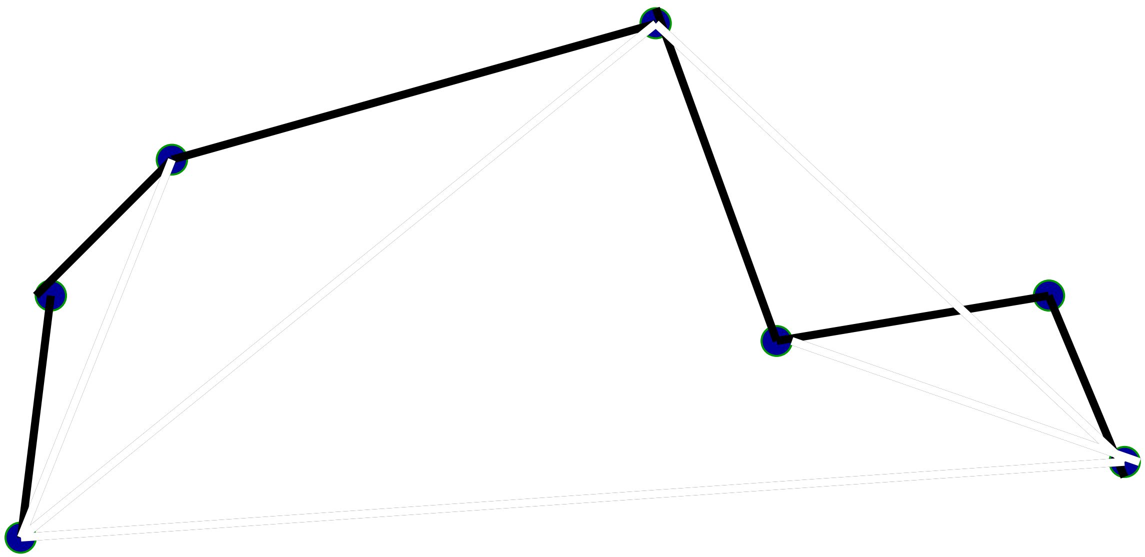
after Sobel filtering

Difficult images

Regions detection

- Regions are bounded by edges
  - detection of closed curves
    - linking all edge points
    - iterative line fitting
    - finding lines or circles or ... with the Hough transform
- Classification
- Region growing algorithm
- Split and Merge algorithm
- Other approaches :
  - snakes, deformable contours, …

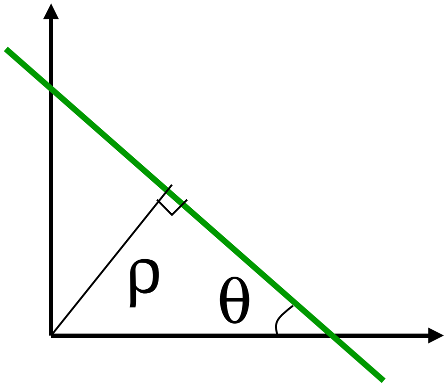
Iterative endpoint fit



Hough transform

- Well known for lines search
- Based on parameterisation of geometrical objects
- Transformation to a parameters space
- Quantification of parameters

Hough transform of a line



- Line parameterisation:

  $\rho = x \cos \theta + y \sin \theta$

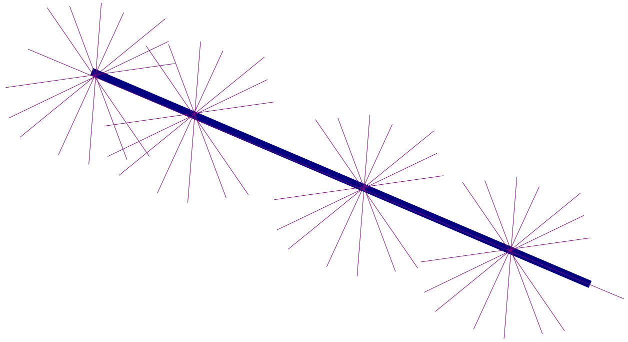
  ou

  $y = a x + b$

- For each point (x,y) labelled as edge pt:
  - for each θ value:
    - compute $\rho = x \cos \theta + y \sin \theta$
    - add a vote for (ρ, θ) : vote[ρ][θ]++;
- Quantification of the parameters space

Why does it works ?
example of a line

- For each point that does not belong to the background, we look for all the lines that go through this point.
- Each founded line vote once.
- If the points are aligned, each point will vote for this line. This line will obtain more votes than any other.

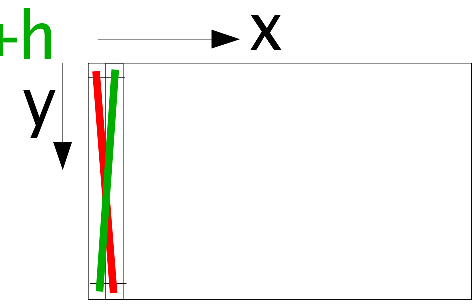


Implementation (1)

- We need to determine the interval of parameter's values :
  - a in $[a_{min}, a_{max}]$
  - b in $[b_{min}, b_{max}]$
- We need to determine the quantification steps (uniform) : Δ a, Δ b
- loop : for($a=a_{min}$; $a <= a_{max}$; a+= Δ a) {...
- uniform quantification of b values

Implementation (2)

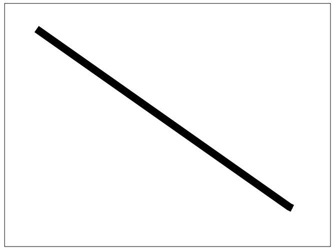
- What interval ?
  - related to the geometrical meaning of the parameters: we do not want to loose lines
  - case $y = ax + b$:
    - the slope varies from - infinite to + infinite
      - we consider values from -h to +h
    - b varies from h(1-w) to h(w+1)



  - case $\rho = x\cos(\theta) + y\sin(\theta)$
    - θ varies from 0 to π
    - ρ varies from -wτο +ρ_{diag} where ρ_{diag} : image diagonal

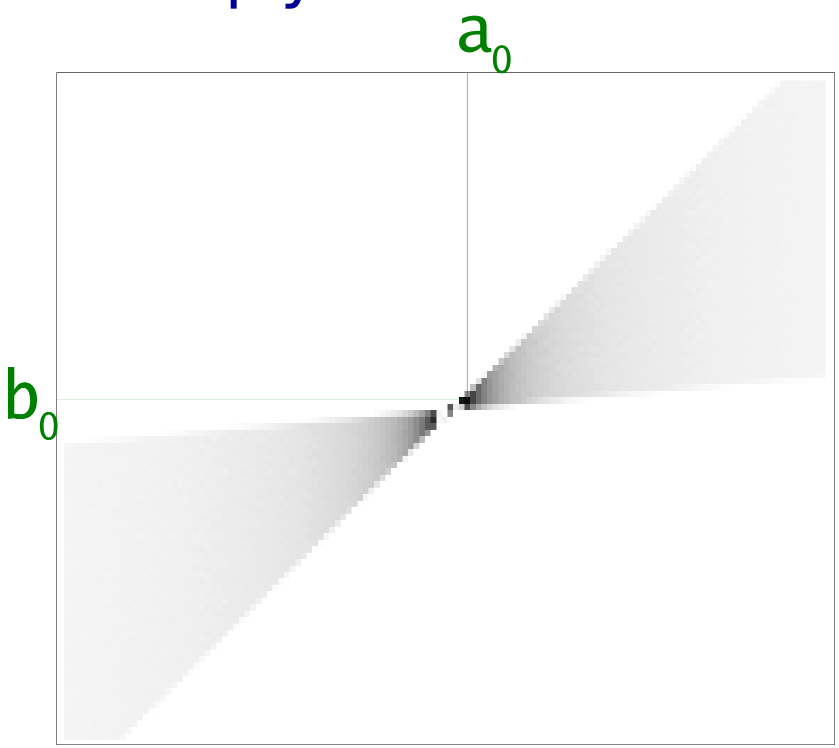
Implementation (3)

- Which step ?
  - the smallest the step, the more accurate the solution but the biggest the image and the highest the computation time.
  - we can deal with 2 stages :
    - large interval, large step
    - small interval, small step
- Hough transform image:
  - of dimensions $(a_{max}-a_{min}+1;b_{max}-b_{min}+1)$
  - to be normalized between 0 and 255 before display
  - score array: indexes often result of an affine transform of real parameters values

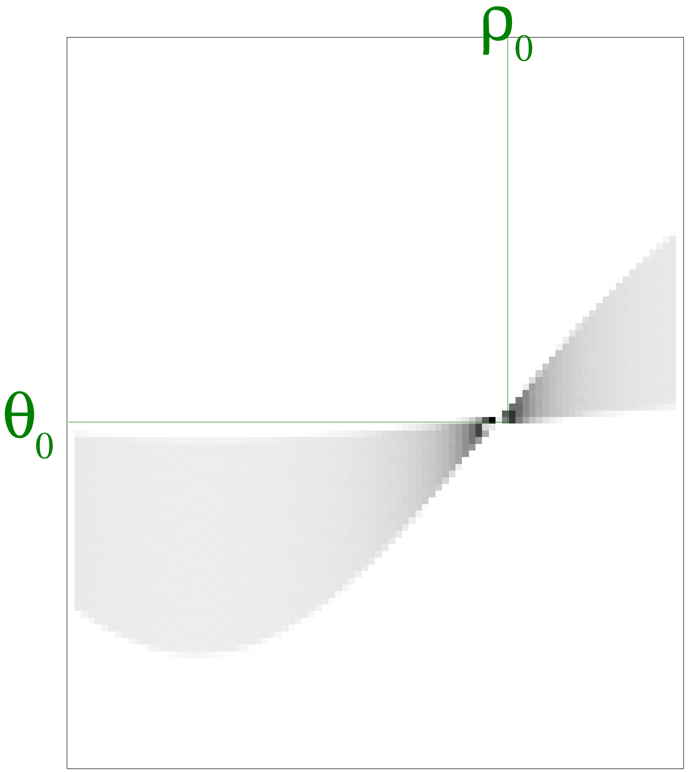


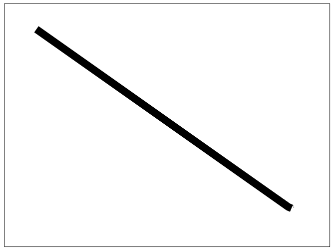
Example of transformed images

parameters space [a,b] t.q. $y=ax+b$

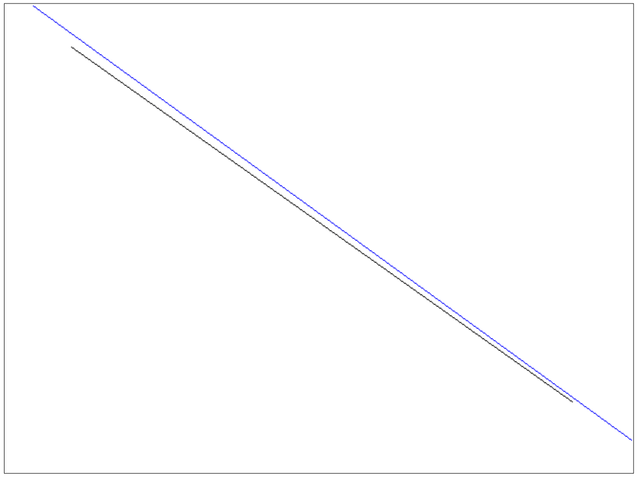


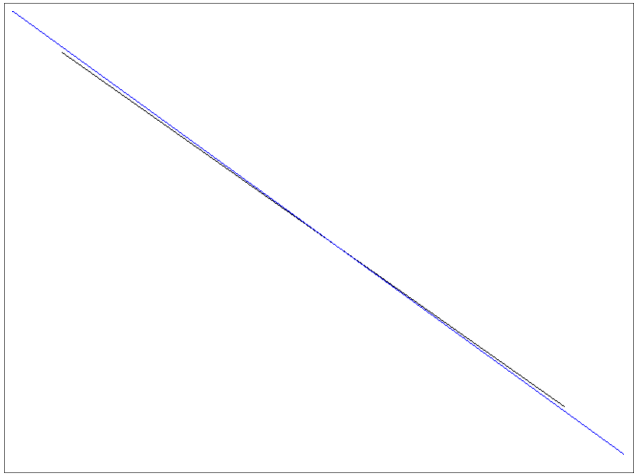
parameters space [ρ,θ] s.as $\rho=x\cos(\theta)+y\sin(\theta)$





Result

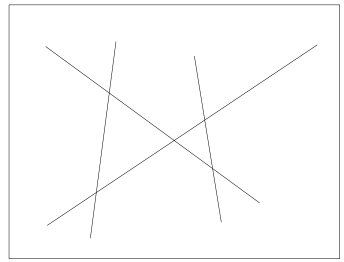




$y = a_0x + b_0$

$\rho_0 = \cos(\theta_0)x + \sin(\theta_0)y$

Another example with 5 lines



Lines detection in an image

- One line detection in an image :
  - finding the point of maximal vote in the Hough transform image
- Multiple lines detection in an image :
  - finding multiple local maxima in the Hough transform image
    - how many maxima ?
    - all points below a threshold ?
- The accuracy of the detection depends on the quantification of the parameters.

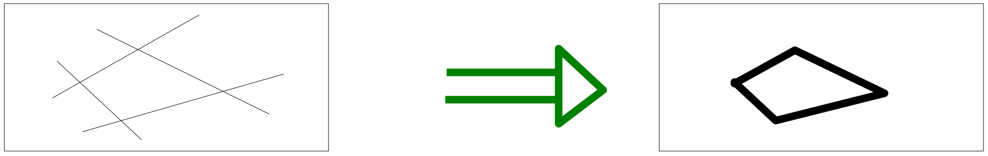
Hough transform of a curve

- Same idea as for a line
- Parameterisation :

  $ax^2 + bxy + cy^2 = 1$
- For each point (x, y) in the image
  - for each value of a between a_{min} and a_{max}
    - for each value of b between b_{min} and b_{max}
      - compute c such as $ax^2 + bxy + cy^2 = 1$
      - ad a vote for (a,b,c): vote[a][b][c]++;

Just try it !

- Compute and display the Hough transform for a line :
  - $y = ax + b$
  - $\rho = \cos(\theta)x + \sin(\theta)y$
- Draw, in the original image, the line that corresponds to the maximal vote
- Find all lines in the image
- Determine the corresponding polygone



Connex components labeling

1	1	1	**0**	1	1	**0**	1
1	1	**0**	1	1	**0**	1	1
1	**0**	1	1	**0**	1	1	1
1	1	1	**0**	1	1	1	1
0	**0**	**0**	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

image after edge detection and thresholding

1	1	1	**0**	2	2	**0**	3
1	1	**0**	4	2	**0**	5	3
1	**0**	6	2	**0**	7	3	3
1	1	1	**0**	8	3	3	3
0	**0**	**0**	9	3	3	3	3
10	10	10	3	3	3	3	3
10	10	10	3	3	3	3	3
10	10	3	3	3	3	3	3

image of labels after the first browsing

1	1	1	**0**	1	1	**0**	3
1	1	**0**	1	1	**0**	3	3
1	**0**	1	1	**0**	3	3	3
1	1	1	**0**	3	3	3	3
0	**0**	**0**	3	3	3	3	3
3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3

image of labels after update

1	2	3	4	5	6	7	8	9	10
			2	3	2	3	3	3	3
	1		1		1				